1. The speed of a 100 metre runner in $\mathrm{ms}^{-1}$ is measured electronically every 4 seconds.

The measurements are plotted as points on the speed-time graph in Fig. 6. The vertical dotted line is drawn through the runner's finishing time.

Fig. 6 also illustrates Model $P$ in which the points are joined by straight lines.


Fig. 6
i. Use Model P to estimate
A. the distance the runner has gone at the end of 12 seconds,
B. how long the runner took to complete 100 m .

A mathematician proposes Model Q in which the runner's speed, $v \mathrm{~ms}^{-1}$ at time $t \mathrm{~s}$, is given by

$$
v=\frac{5}{2} t-\frac{1}{8} t^{2}
$$

ii. Verify that Model Q gives the correct speed for $t=8$.
iii. Use Model Q to estimate the distance the runner has gone at the end of 12 seconds.
iv. The runner was timed at 11.35 seconds for the 100 m .

Which model places the runner closer to the finishing line at this time?
v. Find the greatest acceleration of the runner according to each model.
2. A particle is travelling along a straight line with constant acceleration. $\mathrm{P}, \mathrm{O}$ and Q are points on the line, as illustrated in Fig. 4. The distance from P to O is 5 m and the distance from O to $Q$ is 30 m .


Fig. 4

Initially the particle is at O . After 10 s , it is at Q and its velocity is $9 \mathrm{~ms}^{-1}$ in the direction $\overrightarrow{\mathrm{OQ}}$.
i. Find the initial velocity and the acceleration of the particle.
ii. Prove that the particle is never at $P$.
3. Fig. 1 shows the velocity-time graph of a cyclist travelling along a straight horizontal road between two sets of traffic lights. The velocity, $v$, is measured in metres per second and the time, $t$, in seconds. The distance travelled, $s$ metres, is measured from when $t=0$.


Fig. 1
i. Find the values of $s$ when $t=4$ and when $t=18$.
ii. $\quad$ Sketch the graph of $s$ against $t$ for $0 \leqslant t \leqslant 18$.
4. A car is usually driven along the whole of a 5 km stretch of road at a constant speed of $25 \mathrm{~m} \mathrm{~s}^{-}$ ${ }^{1}$. On one occasion, during a period of 50 seconds the speed of the car is as shown by the speed-time graph in Fig. 7; the rest of the 5 km is travelled at $25 \mathrm{~m} \mathrm{~s}^{-1}$.


Fig. 7
How much more time than usual did the journey take on this occasion? Show your working clearly.
5. Two cars, $A$ and $B$, are travelling in different lanes in the same direction along a straight road.

The initial situation is illustrated in Fig. 5.

- At this time, $A$ is stationary at traffic lights at $O$. The lights have just turned green and $A$ is on the point of moving off.
- $B$ is travelling towards $O$ with speed $20 \mathrm{~m} \mathrm{~s}^{-1}$. $B$ is 75 m behind $A$.

B


Fig. 5
During the subsequent motion,

- A has constant acceleration $2 \mathrm{~m} \mathrm{~s}^{-2}$,
- the traffic lights remain green and B maintains a constant speed $20 \mathrm{~m} \mathrm{~s}^{-1}$.

In order to model the subsequent motion you should make two assumptions.

- The cars can overtake each other with no interference from other traffic.
- The position of a car is defined by a point at its front and so the length of the car need not be considered.
(i) Find the times at which the two cars are side by side.
(ii) Find the distance $A$ travels while it is behind $B$.
(iii) There is a speed camera 400 m from O .

How fast is A travelling when it passes the speed camera?
6. A train is travelling along a straight test track. It starts from rest and reaches its maximum speed after a time of 2 minutes and 21 seconds. During that time it travels 5 km .

Two models, $A$ and $B$, are considered for its motion.
In Model A, it is assumed that the train has constant acceleration.

> (i) Find the acceleration of the train and its maximum speed according to Model A.

In Model B, it is assumed that the acceleration, a $\mathrm{m} \mathrm{s}^{-2}$ at time $t$ seconds after starting, is given by

$$
\mathrm{a}=0.6-3 \times 10^{-5} \times t
$$

(ii) Show that, according to Model B , the time taken for the train to reach its maximum speed is 2 minutes 21.42 seconds (to the nearest 0.01 s ).

Find expressions for the speed of the train and the distance that it has travelled at time $t$, according to Model B.

Hence show that Model B is consistent with the train travelling a distance of 5 km to attain maximum speed.

Find the maximum speed of the train according to this model.
(v) When the train reaches its maximum speed it continues at that speed.

Draw the speed-time graphs for both models on the grid provided, labelling them A and B.

7. A truck is travelling along a straight road ABC , and is slowing down at a constant rate. The truck takes 4 s to travel the 64 m from A to B and it takes another 4 s to travel the 32 m from $B$ to $C$.
(a) Find

- the speed of the truck at A,
the acceleration of the truck.
(b) Find how far beyond $C$ the truck travels before coming to rest.

8. Rory runs a distance of 45 m in 12.5 s . He starts from rest and accelerates to a speed of 4 m $\mathrm{s}^{-1}$. He runs the remaining distance at $4 \mathrm{~m} \mathrm{~s}^{-1}$.

Rory proposes a model in which the acceleration is constant until time $T$ seconds.
(a) Sketch the velocity-time graph for Rory's run using this model.
(b) Calculate $T$.
(c) Find an expression for Rory's displacement at time $t$ s for $0 \leq t \leq T$.
(d) Use this model to find the time taken for Rory to run the first 4 m .

Rory proposes a refined model in which the velocity during the acceleration phase is a quadratic function of $t$. The graph of Rory's quadratic goes through ( 0,0 ) and has its maximum point at $(S, 4)$. In this model the acceleration phase lasts until time $S$ seconds, after which the velocity is constant.
(e) Sketch a velocity-time graph that represents Rory's run using this refined model.
(f) State with a reason whether $S$ is greater than $T$ or less than $T$. (You are not required to calculate the value of $S$.)
9. A bus travelling on a straight road accelerates uniformly from $2.5 \mathrm{~m} \mathrm{~s}^{-1}$ to $7.5 \mathrm{~m} \mathrm{~s}^{-1}$ in 12 s . It then travels at $7.5 \mathrm{~m} \mathrm{~s}^{-1}$ for 20 s before slowing uniformly to rest in 8 s .
(a) Sketch a velocity-time graph for the bus.
(b) Calculate the average speed of the bus.
10. A cyclist is travelling in a straight line. She has a velocity of $3 \mathrm{~ms}^{-1}$ when passing O . After 4 s she reaches A which is 24 m from O . After a further 6 s she reaches B which is 80 m beyond A.

Determine whether modelling the motion as having constant acceleration is consistent with these values.

## Mark scheme




|  |  |  | Motion in 1 Dimension, Kinematics Graphs and Constant Acceleration Formulae models and there were many correct answers, well presented with ciear statements as to which model wasbeing considered. |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Total | 18 |  |
| 2 | i | Ether $s=\frac{1}{2}(u+v) t_{\text {Take } \mathrm{O} \text { as the origin. }}$ | M1 | Use of one relevant equation, including substitution |
|  | i | $30=\frac{1}{2} \times(u+9) \times 10$ |  | Enter text here. |
|  | i | $u=-3$ | A1 |  |
|  | i | $v=u+a t$ | M1 | Use of a second relevant equation including substitution |
|  | i | $9=-3+10 a$ |  |  |
|  | i | $a=1.2$ | ${ }^{\text {A }}$ |  |
|  | i | or $v=u+a t \Rightarrow u+10 a=9$ | m1 | Use of one relevant equation, including substitution |
|  | i | $s=u t+\frac{1}{2} a t^{2} \Rightarrow u+5 a=3$ | M1 | Use of a second relevant equation including substitution |
|  | i | Solving simutaneously: $a=1.2$ | A1 |  |
|  | i | $u=-3$ | ${ }^{\text {A }}$ |  |
|  | i | $\text { or } s=v t-\frac{1}{2} a t^{2}$ | M1 | Use of one relevant equation, including substitution |
|  | i | $\Rightarrow a=1.2$ | A1 |  |
|  | i | +at | M1 | Use of a second relevant equation including substitution |
|  |  |  |  | Examiner's Comments |
|  | i | $\Rightarrow u=-3$ | A1 | This question involved a particle traveling under constant acceleration along a straight line, two constant acceleration formulae were required and most candidates obtained the right answers but there were some careless mistakes, and some mistakes in quoting the standard results. |


3 (


\begin{tabular}{|c|c|c|c|c|}
\hline 5 \& \& \begin{tabular}{l}
A: \(x=t\) \\
B: \(x=-75+20 t\) \\
When the cars are side by side, \(\ell=-75+20 t\)
\[
\begin{aligned}
\& t=-20 t+75=0 \\
\& (t-5)(t-15)=0
\end{aligned}
\] \\
The times are 5 seconds and 15 seconds
\end{tabular} \& B1
B1
M1
M

A1

[4] \& | on in 1 Dimension, Kinematics Graphs and Constant Acceleration Formulae Or displacements from B's start point: $x=f+75 \text { and } x=20 t$ |
| :--- |
| Must be consistent |
| For equating two distances even if inconsistent |
| Examiner's Comments |
| This question involved the motion of two cars in parallel lanes on a road. One had constant acceleration and the other travelled at constant speed. One car was initially behind the other. |
| In part (i) candidates were asked to find the times when the cars were side by side. While there were many fully correct answers to this, there were also plenty of sign errors involving the 75 m difference in starting position. A few candidates did not realise that answering this question involved setting up an equation for the time $t$. | <br>

\hline \& ii \& | For A, $s=225$ when $t=15$ <br>  $s=25$ when $t=5$ |
| :--- |
| So $A$ is behind $B$ for 200 m |
| Alternative Using motion of $B$ |
| Speed of B is (constant at) $20 \mathrm{~m} \mathrm{~s}^{-1}$ |
| So between $t=5$ and $t=15$, B travels $20 \times(15-5)(=200 \mathrm{~m})$ | \& M1

A1
[2]

M1 \& | FT for two positive times from part (i) for the M mark only |
| :--- |
| Both values of s attempted |
| CAO |
| Or equivalent, eg using $s=u t+\frac{1}{2} a t^{2}$ |
| with $a=0$ | <br>

\hline
\end{tabular}




\begin{tabular}{|c|c|c|c|}
\hline \& \begin{tabular}{l}
At maximum speed the acceleration is zero
\[
t=\sqrt{\frac{0.6}{3 \times 10^{-5}}}(=\sqrt{20000})=141.421 \ldots
\] \\
So 2 minutes 21.42 seconds
\end{tabular} \& \multicolumn{2}{|l|}{\begin{tabular}{l}
Motion in 1 Dimension, Kinematics Graphs and Constant Acceleration Formulae Setting \(a=0\) in the given equation for \(a\). \\
Accept answer in seconds \\
Examiner's Comments \\
The question then moved on to a model with variable acceleration. Part (ii) required candidates to recognise that when the train reached maximum speed its acceleration is zero and so obtain a given value for the time taken. While most candidates were successful in this, a substantial number did not recognise the significance of zero acceleration and tried other unsuccessful approaches, often involving a lot of fruitless work.
\end{tabular}} \\
\hline \& \begin{tabular}{l} 
Integrating \\
\begin{tabular}{l}
\(V=0.6 t-0.00001 t^{\beta}\) \\
\((+C)\),
\end{tabular} \\
\hline \\
\(s=0.3 t-0.0000025 t^{( }(+A)\) \\
\(t=0, s=0 \Rightarrow k=0\)
\end{tabular} \& M1
A1
A1
A1

A1

A \& | Attempt at integration. |
| :--- |
| Or equivalent, eg $v=0.6 t-10^{-5} \times \beta(+c)$ |
| Coefficients do not need to be simplified in either integral. |
| FT from $v$. Integration must be attempted. |
| Or equivalent, eg $s=0.3 t-2.5 \times 10^{-6} \times t^{A}(+A)$ |
| Use of mechanics and not assertion to show $k=0$ |
| Examiner's Comments |
| Part (iii) required candidates to integrate the acceleration to find the speed and then to integrate again to find the distance travelled by the train. Many candidates did not consider the constants of integration, or just declared them to be zero without any reason, and this was penalised. | <br>

\hline \& | Substituting $t=141.42 \ldots$ in $s=0.3 t^{2}-0.0000025 t^{4}$ |
| :--- |
| $s=5000$ so consistent with 5 km |
| Substituting $t=141.42 \ldots$ in $v=0.6 t-0.00001 t^{\beta}$ $v=56.57 \mathrm{~ms}^{-1}$ | \& M1

A1 \& | Allow substituting $s=5000$ to show that $t=141.42 \ldots$ |
| :--- |
| Notice that $141.42 \ldots=\sqrt{20000}$ and so the answer of 5000 is exact | <br>

\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|c|}
\hline \& \& \& \multicolumn{2}{|l|}{\begin{tabular}{l}
Motion in 1 Dimension, Kinematics Graphs and Constant Acceleration Formulae \\
Examiner's Comments \\
In part (iv) candidates were expected to use the time given in part (ii) and their expression for the distance travelled from part (iii) to verify the distance the train had travelled in attaining maximum speed. Many knew just what to do and were successful. Some tried to do the question in reverse, substituting the distance and forming a quartic equation for the time; this was a viable approach and a few candidates realised that their equation could be written as a quadratic in \(t 2\) and went on to solve it. \\
The question continued to ask for the maximum speed and this was well answered, even by those who had not been successful with the distance.
\end{tabular}} \\
\hline \& \&  \& B1

B1
B1
B1
B1

[4] \& | Vertical scale from 0 that ensures that at least half the height is used. |
| :--- |
| For the remaining marks do not allow FT from earlier parts. |
| A: A straight line from $(0,0)$ to $(141,70.9)$ |
| B: A curve from $(0,0)$ to $(141,56.6)$; the curvature must be in the right sense. |
| Both A from $(141,70.9)$ to $(160,70.9)$ and $B$ from $(141,56.6)$ to $(160,56.6)$. CAO. |
| Examiner's Comments |
| The final part (v) required the two models to be shown on a speed-time graph. This produced a wide spread of marks. Most candidates knew what they were trying to do but made errors. Some lost a mark by not showing the motion after the train had reached maximum speed and many others drew a straight line rather than a curve for the variable acceleration model. | <br>

\hline \& \& Total \& 18 \& <br>

\hline 7 \& a \& | For AB: use of $s=u t+\frac{1}{2} a t^{2}$ with $s=64$, |
| :--- |
| $t=4$ gives | \& | M1(AO3.3) |
| :--- |
| A1(AO1.1b) | \& Allow use of a different sign convention (egnegative a) provided it is used consistently throughout and is explained <br>

\hline
\end{tabular}

$64=4 u+\frac{1}{2} a \times 4^{2}$

For AC: $s=96$ and $t=8$
$96=8 u+\frac{1}{2} a \times 8^{2}$

Solve simultaneously ( $16=u+2 \mathrm{a}, 12=u+4 \mathrm{a})$
$u=20$ and $a=-2$

Alternative solution

| For AB: use of | $s=u t+\frac{1}{2} a t^{2}$ | with $s=64$, |
| :--- | :--- | :--- |

$t=4$ gives
$64=4 u+\frac{1}{2} a \times 4^{2}$

For $B C$ : speed at $B$ is $u+4 a$
$32=4(u+4 a)+\frac{1}{2} a \times 4^{2}$

Solve simultaneously ( $16=u+2 \mathrm{a}, 8=u+6 \mathrm{a})$
$\mathrm{u}=20$ and $\mathrm{a}=-2$

For both 96 and 8 seen

M1 (AO1.1a)
Forming second equation in $u$ and $a$

May be implied if calculator
M1

A1

B1

M1

M1

A1
[6]
used

Both correct; allow deceleration $=2$

Correct (unsimplified) equation
Use of $v=u+a t$ for AB
Forming second equation in $u$ and $a$
oe BC

Both correct; allow deceleration $=2$

Allow use of a different sign convention (eg negative a) provided it is used consistently throughout and is explained

|  | b | $\begin{aligned} & u=20, v=0, a=-2 \text { gives } \\ & 0=20^{2}-2 \times 2 \times s \end{aligned}$ <br> $s=100$ so truck comes to rest 4 m beyond C | M1 (AO3.4) <br> Motion in 1 Dimension, Kinematics Graphs and Constant Acceleration Formulae |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | A1(AO1.1b) | Allow for any suvat equation(s) leading to a value for $s$ |  |
|  |  |  | [2] |  |  |
|  |  | Total | 8 |  |  |
|  |  | ${ }_{4}{ }^{\text {velocity }}$ | $\begin{gathered} \mathrm{B} 1 \\ \text { (AO1.1a) } \end{gathered}$ | Two line segments with one horizontal <br> ( $T, 4$ ) and $(12.5,4)$ labelled or indicated on scales. Allow their 2.5 marked instead of $T$. On axes labelled $v$ and $t$ oe |  |
|  |  |  | (AO1.1a) | Examiner's Comments <br> Most candidates gave a graph with two straight line not fully labelled. <br> Make sure you label the axes and sh | ments but marks were often lost for graphs that were <br> the values of $v$ and $t$ at the significant points. |
|  | b | $\frac{1}{2} \times 4 \times(12.5+(12.5-T))=45$ | $\begin{gathered} \text { M1 } \\ \text { (AO3.1a) } \end{gathered}$ | Attempt to find area of trapezium or both the thetriangle $\left(\frac{1}{2} T \times 4\right)$ | Suvat equations can be used for two phases of motion. |



(A)



|  | For OA: $24=3 \times 4+\frac{1}{2}_{a \times 4^{2}}$ $a=1.5$ <br> For OB: $s=3 \times 10+\frac{1}{2} \times 1.5 \times 10^{2}$ $O B=105 \mathrm{~m}$ <br> Actual distance is 104 m , which is very close, so constant acceleration is a good model | M1 A1 M1 M A1 A1 | nin 1Dimension, Kinematics Graphs a <br> Find a for OA using $u=3, s=$ $24, t=4$ <br> Use of $a=1.5$ for OB , oe <br> Clear comparison and conclusion <br> Allow alternative conclusion, i.e. that the model is not [exactly] consistent with the data | dConstant Acceleration Formulae Allow credit for any complete method <br> Do not allow $u=3$ as initial speed for AB |
| :---: | :---: | :---: | :---: | :---: |
|  |  | [5] |  |  |
|  | Total | 5 |  |  |

